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D-Branes from N D(-1)-branes in bosonic and type-IIA string theory

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Abstract: In this paper we would like to discuss the emergence of D-branes from infinite many D-instantons in bosonic and type-IIA string theory in the framework of boundary string field theory.

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Contents

1. Introduction

The boundary String Field Theory (BSFT) [1]–[5][1] is a version of open string field theory in which the classical configuration space is a space of two-dimensional world-sheet theories on the disk which are conformal in the interior of the disk and which have arbitrary boundary actions. The boundary conformal theories correspond to the solutions of classical equations of motion. In the recent papers [6]–[19] (For the world-sheet approach to this problem, see [21, 22, 23]) the BSFT was used in the analysis of the tachyon condensation on the unstable D-brane systems in string theories (For review and extensive list of references, see [24]). It was shown that BSFT is very effective tool for the study of this problem. In particular, it was shown that:

- The condensation to the closed string vacuum and to lower dimensional branes involves excitations of only one mode of string field-tachyon.
- The exact tachyon potential can be computed in BSFT and its qualitative features agree with the Sen conjecture [25].
- The exact tachyon profiles corresponding to the tachyon condensation to the lower dimensional D-brane give rise to descent relations between the tensions of various branes [14] which again agree with those expected [25].

The problem of tachyon condensation was also studied from the point of view of Witten's open string field theory [26]–[46]. In contrast with BSFT, the tachyon condensation in general involves giving the expectation values of infinite number of component fields. As a consequence, only approximate results are available.

[1] In this paper we use the name of the theory proposed in [18].

In this note we would like to study the problem of the tachyon condensation on the system of N D-instantons in the limit $N \to \infty$ in the framework of BSFT. This system has been studied in the low energy approximation in [48] and in the language of the Witten's open string field theory in [49]. The problem of the tachyon condensation on the system of D0-branes has been also studied in [50] from the point of view of the action obtained from BSFT, where the abelian action given in [7, 11] was generalised to the non-abelian case.

The results presented in this paper can be considered as an additional support for the approach given in [50]. More precisely, in this paper we will study the problem of the tachyon condensation in system of N D-instantons in bosonic and type-IIA theory[2] in limit $N \to \infty$. We will see that the problem of the tachyon condensation is again extremely simple from the point of view of BSFT. In particular, we will calculate the partition sum on the disk, which is basic ingredient of BSFT, for configurations of D-instantons corresponding to higher dimensional D-branes (For review of the construction of higher dimensional D-branes from the lower dimensional ones, see [52]–[54]) with the tachyon field turned on. This approach is similar to the study of the systems of N D-instantons in the boundary state formalism [55]–[58]. We hope that our approach can bring new facts about the tachyon condensation on the system of N D-instantons and emergence of all D-branes from the lower dimensional ones. In particular, we hope that this approach could have some relation to the Matrix theory.

The organisation of the paper is as follows. In section (2) we give a brief review of BSFT. In section (3) we present the calculation of the partition function for D-instantons in the bosonic string theory. In particular, we will show that with the background configuration of N D-instantons well known from the matrix theory [52, 53] we are able to obtain all even dimensional D-branes from the point of view of BSFT. Then we extend this approach to the case of odd dimensional D-branes. However, to find such a solution we should turn on the tachyon mode in the BSFT and study its condensation in the same way as in [7, 11].

In section (4) we will analyse the same problem in the superstring BSFT, using the recent proposal [11] relating BSFT action and the superstring partition sum on the disk. We will again see that from the system of N D-instantons we obtain all D-branes in type-IIA theory. In conclusion (5) we outline our results and suggest possible extension of our work.

2. A brief review of the boundary string field theory

In this section we review the basic facts about boundary string field theory [1]. A general string field configuration in the boundary string field theory (or background independent string field theory, in any case BSFT) is associated with a boundary

[2] In the same way we could study unstable D0-branes in type-IIB theory.

operator of ghost number one in the world-sheet theory of matter and ghost system. We will take the world-sheet to be a disk with unit radius and the metric

$$ds^2 = d\rho^2 + \rho^2 d\tau^2 \,, \qquad 0 < \rho < 1 \,, \qquad \tau \in \langle 0, 2\pi \rangle \,. \tag{2.1}$$

If $\{\mathcal{O}_I\}$ denotes a set of boundary vertex operators of ghost number 1, we can expend a general operator $\mathcal{O}$ of ghost number 1 as

$$\mathcal{O} = \sum_I \lambda^I \mathcal{O}_I \,. \tag{2.2}$$

We shall restrict to the operators of the form $\mathcal{O} = c\mathcal{V} = \sum_I c\lambda^I \mathcal{V}_I$ where c is the ghost field and $\mathcal{V} = \sum_I \lambda^I \mathcal{V}_I$ is a boundary operator in matter theory. In this case, a string field configuration associated with the operator $c\mathcal{V}$ is described by the world-sheet action

$$S = S_{\mathrm{Bulk}} + \int_0^{2\pi} \frac{d\tau}{2\pi} \mathcal{V}(\tau) \,, \tag{2.3}$$

where S_{Bulk} denotes the bulk world-sheet action corresponding to the closed string background. When we restrict to the trivial background, then S_{Bulk} describes of conformal field theory (CFT) of free 26 scalar fields X^{μ}[3] and the (b, c) ghost system. For non-abelian case, the boundary interaction $\mathcal{V}$ is promoted to $N \times N$ matrix and the path integral measure on the disk is weighted with

$$e^{-S_{\mathrm{Bulk}}} \,\mathrm{Tr}\, P \exp\left(- \int_0^{2\pi} \frac{d\tau}{2\pi} \mathcal{V} \right) . \tag{2.4}$$

For such configurations, the bosonic string field theory action $S(\lambda^I)$ is obtained as a solution of equation

$$\frac{\partial S}{\partial \lambda^I} = \frac{K}{2} \int \frac{d\tau}{2\pi} \int \frac{d\tau'}{2\pi} \left\langle \mathcal{O}_I(\tau) \{ Q_B, \mathcal{O}(\tau') \} \right\rangle_{\mathcal{V}} \,, \tag{2.5}$$

where $\langle \cdots \rangle_{\mathcal{V}}$ denotes correlation function in the world-sheet field theory described by the action (2.3), Q_B is BRST charge and K is normalisation constant which has been fixed in [8] as

$$K = T_p \,, T_p = \frac{2\pi}{(4\pi^2 \alpha')^{(p+1)/2}} \,. \tag{2.6}$$

It is clear that (2.5) determines action up to additive constant.

It was also shown in [1, 2, 4, 5] that the action (2.5) is related to the partition sum on the disk $Z(\lambda)$ via

$$S = \left(\sum_I \beta^I \frac{\partial}{\partial \lambda^I} + 1 \right) Z(\lambda) \,. \tag{2.7}$$

[3]In this paper we consider the space-time with euclidean signature and the metric $g_{\mu\nu} = \delta_{\mu\nu}$.

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This definition also fix the ambiguity in the definition of the action by the requirement that at the fix points of the boundary RG (at which $\beta(\lambda^*) = 0$)

$$S(\lambda^*) = Z(\lambda^*)\,. \tag{2.8}$$

These results corresponds to abelian U(1) case. The generalisation to the non-abelian case seems to be nontrivial task thanks to the presence of the path-ordered product in the boundary interaction terms. Instead of address this problem in this paper, we will proceed in slightly different way. The main point of our calculation will be the partition sum on the disk that, as we have seen, plays the prominent role in the abelian case. We will calculate the partition sum for different configurations of D-instantons and we will show that it corresponds to the partition sum for specific D-brane in particular background. Then we use well known results from the abelian case to obtain the correct tension of the resulting D-brane. The correctness of our analysis will be especially seen in the supersymmetric case where it was conjectured [11] that the partition sum is equal to the space-time action. It would be certainly nice to obtain non-abelian action directly from BSFT theory, especially in bosonic case. We hope to return to this problem in the future.

In the next section we will calculate the partition sum in the bosonic theory for N D-instantons and we will show that from this configuration all D-branes in bosonic string theory naturally emerge.

3. D-instantons in BSFT

In this section we would like to show that through the configuration of N D(-1)-branes (D-instantons) in the limit $N \to \infty$ it is possible to obtain all D-branes of even and odd dimensions from the BSFT theory. We begin with the construction of even dimensional D-branes in bosonic string theory. The starting point is the evaluation of the partition sum on the disk

$$Z = \left\langle \mathrm{Tr}\, P \exp\left(i \int_0^{2\pi} d\tau \Phi_I \partial_n X^I(\tau) \right) \right\rangle, \tag{3.1}$$

where τ is a coordinate labelling the boundary of the disk and P is the path-ordering defined as

$$P e^{\int d\tau M(\tau)} = \sum_{N=0}^{\infty} \int d\tau^1 \cdots d\tau^N \theta(\tau_{12}) \cdots \theta(\tau_{N-1,N}) M(\tau_1) \cdots M(\tau_N)\,, \tag{3.2}$$
$$\theta(\tau_{12}) = \theta(\tau_1 - \tau_2)\,.$$

In (3.1) Φ_I, $I = 1, \ldots, 26$ are hermitean matrices describing the background configuration of N D-instantons and $X^I(\tau)$ is a string field living on the boundary of the world-sheet and ∂_n is a normal derivative to the boundary of the world-sheet. In the following we will always presume the limit $N \to \infty$.

Let us consider the background configuration

$$\Phi_a = \Phi_a\,, \quad [\Phi_a, \Phi_b] = i\theta_{ab}\,, \quad a, b = 1, \ldots, 2p, \quad \Phi_\alpha = 0\,, \quad \alpha = 2p+1, \ldots, 26\,. \tag{3.3}$$

We will show that this configuration corresponds to D$(2p-1)$-brane siting in the origin of the transverse space labelled with coordinates x^α, $\alpha = 2p+1, \ldots, 26$ and with the the gauge field background turned on its world-volume with the constant field strength $F_{ab} = \theta_{ab}$.

From (3.3) we see that Φ_a are equivalent to the quantum mechanics operators with nontrivial commuting relations. Then we can use the well known relation between the trace over Hilbert space and the path integral

$$\begin{aligned} \mathrm{Tr}\, P \exp\left(-i \int d\tau H(\tau)\right) &= \int \langle q| \exp\left(-i \int d\tau H(\tau)\right) |q\rangle \\ &= \int [dq][dp] \exp\left(i \int d\tau \left[p(\tau)\dot{q}(\tau) - H(p(\tau), q(\tau))\right]\right), \end{aligned} \tag{3.4}$$

with $\dot{q} = \partial_\tau q$ and where p is a momentum conjugate to q. In our case, the hamiltonian is

$$H(\tau) = -\Phi_a \partial_n X^a(\tau)\,. \tag{3.5}$$

with the operators Φ_a. Then we can rewrite the trace in (3.1) as a path integral

$$\int \prod_{a=1}^{2p} [\phi_a] \exp\left(i \int d\tau \left(\frac{1}{2}\phi_a(\tau)\theta^{ab}\dot{\phi}_b(\tau) + \phi_a(\tau)\partial_n X^a(\tau)\right)\right), \tag{3.6}$$

with $\theta_{ac}\theta^{cb} = \delta_a^b$. We can easily perform the integration over ϕ. Firstly, the kinetic term is equal to

$$\begin{aligned} \frac{i}{2}\int d\tau \phi_a(\tau)\theta^{ab}\dot{\phi}_b(\tau) &= \frac{1}{2}\int d\tau d\tau' \phi_a(\tau') i\delta(\tau' - \tau)\theta^{ab}\dot{\phi}_b(\tau) \\ &= -\frac{1}{2}\int d\tau d\tau' \phi_a(\tau') i\partial\delta(\tau' - \tau)\theta^{ab}\phi_b(\tau)\,, \quad \partial = \partial_\tau\,, \quad \partial' = \partial_{\tau'}\,, \end{aligned} \tag{3.7}$$

so that the path integral is equal to (up to numerical factor)

$$\begin{aligned} &\int [d\phi^a] \exp\left(-\int d\tau d\tau' \left[\frac{1}{2}\phi_a(\tau')\Delta(\tau', \tau)^{ab}\phi_b(\tau) - \phi_a(\tau) i\partial_n X^a(\tau)\right]\right) = \\ &= \exp\left(-\frac{1}{2}\int d\tau d\tau' \partial_n X^a(\tau)\Delta(\tau, \tau')^{-1}_{ab}\partial_n X^b(\tau')\right), \quad \Delta(\tau', \tau)^{ab} = i\partial\delta(\tau' - \tau)\theta^{ab}\,. \end{aligned} \tag{3.8}$$

To proceed further we must calculate Δ^{-1} that should obey

$$\int_0^{2\pi} d\tau'' \Delta^{-1}(\tau - \tau'')\Delta(\tau'' - \tau') = \delta(\tau - \tau') - \frac{1}{2\pi}, \; \delta(\tau - \tau') = \frac{1}{2\pi}\sum_n e^{in(\tau - \tau')}\,. \tag{3.9}$$

From this definition we easily obtain

$$\Delta(\tau',\tau)^{ab} = \theta^{ab}\frac{1}{2\pi}\sum_n n e^{in(\tau'-\tau)} \Rightarrow \Delta(\tau,\tau')^{-1}_{ab} = \theta_{ab}\frac{1}{2\pi}\sum_n \frac{1}{n} e^{in(\tau-\tau')} . \tag{3.10}$$

We expand the string field as

$$X^a(\tau,\rho) = \sqrt{\frac{\alpha'}{2}} \sum_{n=-\infty, n\neq 0}^{\infty} \rho^n X^a_n e^{in\tau} , \qquad \partial_n X^a(\tau,\rho=1) = \sqrt{\frac{\alpha'}{2}} \sum_{n=-\infty, n\neq 0}^{\infty} n X^a_n e^{in\tau} . \tag{3.11}$$

Note that there is no zero mode thanks to the Dirichlet boundary conditions. Then (3.8) is equal to

$$-\frac{1}{2}\int d\tau d\tau' \partial_n X^a(\tau)\Delta(\tau,\tau')^{-1}_{ab}\partial_n X^b(\tau') = \alpha'\pi\sum_{m=1}^{\infty} m\theta_{ab}X^a_{-m}X^b_m , \tag{3.12}$$

that agrees precisely with the expression [18]

$$S = \frac{i}{2}\int_0^{2\pi} d\tau F_{ab}X^a(\tau)\dot{X}^b(\tau) = \pi\alpha'\sum_{n=1}^{\infty} n F_{ab}X^a_{-n}X^b_n , F_{ab} = \theta_{ab} , \tag{3.13}$$

arising from the term

$$S = -i\int_0^{2\pi} d\tau A_a(X^a)\partial_\tau X^a(\tau) . \tag{3.14}$$

in case of the constant field strength. As a result, we have obtained the partition sum on the disk for a D$(2p-1)$-brane in the presence of the gauge field F_{ab}. Since the gauge field corresponds to the marginal operator, the partition sum is equal to space-time action that is the Dirac-Born-Infeld action [21, 59]

$$S = T_{2p-1}\int d^{2p}x\sqrt{\det(\delta_{ab}+2\pi\alpha' F_{ab})} . \tag{3.15}$$

We must mention that the construction presented above is an analogue of the construction of even dimensional D-branes from the low-energy action for N D-instantons [47, 48] based on the Matrix theory (for a nice review, see [52, 53]).

In order to describe odd dimensional D-brane we should include the tachyon into the boundary interaction term for N D-instantons and study its tachyon condensation. Let us include this matrix valued tachyon field. In this case the boundary action has a form

$$S_{\text{bound}} = \frac{1}{2\pi}\int d\tau T - i\int d\tau \Phi_a\partial_n X^a(\tau) = -i\int d\tau\left(\frac{i}{2\pi}T + \Phi_a\partial_n X^a(\tau)\right) , \tag{3.16}$$

where T does not depend on the world-sheet field $X(\tau)$ as it should be for D-instanton background. In order to obtain D$(2p-2)$-brane we propose the tachyon field in the form

$$T = u(\Phi_2)^2\,, \tag{3.17}$$

where Φ are given as in (3.3). For simplicity we consider the matrix θ_{ab} in the form

$$\theta_{ab} = \begin{pmatrix} 0 & \theta & 0 \\ -\theta & 0 & 0 \\ 0 & 0 & \theta_{ij} \end{pmatrix}, \qquad i,j = 3,\dots,2p\,. \tag{3.18}$$

In this case the partition sum is equal to

$$\begin{aligned} Z &= \langle e^{-S_{\mathrm{bound}}} \rangle \\ &= \left\langle \mathrm{Tr}\, P \exp\left(i \int d\tau \left[\frac{i}{2\pi} u(\Phi_2)^2 + \Phi_a \partial_n X^a(\tau) \right] \right) \right\rangle \\ &= \langle \mathrm{Tr}\, P \exp(-i \int_0^{2\pi} d\tau H(\tau)) \rangle\,. \end{aligned} \tag{3.19}$$

As in the case of pure gauge field we rewrite this expression using the path integral formalism. The factor $\Delta(\tau-\tau')^{ij}$ is the same as in the case of the boundary interaction without tachyon studied above and for $x,y = 1,2$ we obtain from the expression

$$\begin{aligned} i\int d\tau L(\tau) &= i \int d\tau \left(\frac{1}{2}\phi_x(\tau)\theta^{xy}\dot{\phi}_y(\tau) + \frac{i}{2\pi}u(\Phi_2(\tau))^2 + \partial_n X^x(\tau)\phi_x(\tau) \right) \\ &= -\int d\tau d\tau' \left(\frac{1}{2}\phi_x(\tau')\Delta(\tau',\tau)^{xy}\phi_y(\tau) - i\phi_x(\tau)\partial_n X^x(\tau) \right), \quad x,y = 1,2\,, \\ \Delta(\tau',\tau)^{xy} &= i\partial\delta(\tau'-\tau)\theta^{xy} - \frac{1}{\pi}u\delta(\tau'-\tau)\delta^{2x}\,, \end{aligned} \tag{3.20}$$

or more precisely

$$\begin{aligned} \Delta(\tau'-\tau)^{xy} &= \frac{1}{2\pi}\sum_n \left(n\theta^{xy} - \frac{u}{\pi}\delta^{2x} \right) e^{in(\tau-\tau')} = \sum_n E_n^{xy} e^{in(\tau-\tau')}\,, \\ E_n^{xy} &= \begin{pmatrix} 0 & \frac{1}{\theta}n \\ -\frac{1}{\theta}n & -\frac{1}{\pi}u \end{pmatrix}, \\ (E_n)^{-1}_{xy} &= \frac{\theta^2}{n^2}\begin{pmatrix} -\frac{1}{\pi}u & -\frac{n}{\theta} \\ \frac{n}{\theta} & 0 \end{pmatrix}, \\ \Delta^{-1}(\tau-\tau')_{xy} &= \frac{1}{2\pi}\sum_m (E_m)^{-1}_{xy} e^{im(\tau-\tau')}\,, \end{aligned} \tag{3.21}$$

so that the partition sum corresponding to X^x, $x = 1,2$ is equal to

$$Z = \left\langle \exp\left(-\frac{1}{2}\int d\tau d\tau' \partial_n X^x(\tau)\Delta^{-1}(\tau-\tau')_{xy}\partial_n X^y(\tau') \right) \right\rangle, \tag{3.22}$$

where we have

$$-\frac{1}{2}\int d\tau d\tau' \partial_n X^x(\tau)\Delta^{-1}(\tau-\tau')_{xy}\partial_n X^y(\tau') =$$
$$= \frac{\alpha'\pi}{2}\sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty}\left(m\theta_{xy}X^x_{-m}X^y_m - \frac{\theta^2}{\pi}X^1_{-m}uX^1_m\right). \qquad (3.23)$$

We see that the second term above corresponds to

$$-\frac{\alpha'}{2}u\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty}X^1_{-n}X^1_n = -\frac{1}{2\pi}\int_0^{2\pi} d\tau u X^1(\tau)X^1(\tau)\,, \qquad (3.24)$$

where we have made a substitution

$$u\theta^2 \to u\,, \qquad (3.25)$$

and where we have used the fact that in the original instanton model the zero modes X^a_0 are missing. When we combine the previous expression with the gauge field term $A_i\partial_\tau X^i(\tau)$ whose emergence is the same as in the pure gauge field case, we obtain the partition sum on the disk in the well known form

$$Z = \langle e^{-S_{\rm bound}}\rangle = \left\langle \exp\left(-\int_0^{2\pi} d\tau\left[\frac{1}{2\pi}u(X^1(\tau))^2 - iA_a(X(\tau))\partial_\tau X^a(\tau)\right]\right)\right\rangle. \qquad (3.26)$$

Now it is easy to find the value of the action for the resulting object, following the recent results [7, 10, 12]. For our purpose we will closely follow [12]. We must also stress that thanks to the form of the boundary interaction (3.26) which corresponds to D$(2p-1)$-brane with the background gauge field, we should include the zero mode of the string field $X^a(\tau)$ into our analysis since we have effectively replaced the Dirichlet boundary conditions with the boundary conditions appropriate for the study of the tachyon condensation on the D$(2p-1)$-brane.

It is clear that the gauge field F_{ij} decouples from the tachyon term and consequently contributes to the partition function with the factor

$$\int d^{2p-2}x\sqrt{\det(\delta_{ij}+2\pi\alpha' F_{ij})}\,. \qquad (3.27)$$

The contribution from the $x,y=1,2$ sector is almost the same as in [12]

$$\int dx^2\sqrt{v}e^{\gamma v}e^{-a}\Gamma(v)\,, v = \frac{1}{1+(2\pi\alpha'\theta)^2}2\alpha' u\,, \qquad (3.28)$$

where the integral over x^2 is a consequence of the existence of the zero mode in $X^2(\tau)$. In the previous expression we have also included the constant part of the tachyon a which would correspond in the original matrix model to the term $a1_{N\times N}$. The whole partition function is equal to

$$Z(v,a,F)=Ke^{-a}\sqrt{v}e^{\gamma v}\Gamma(v)\int d^{2p-1}x\sqrt{\det(\delta_{ij}+2\pi\alpha' F_{ij})}\,, \tag{3.29}$$

where K is a numerical factor which will be determined in a moment. As was shown in [3, 7, 10, 12] the space-time action is then equal to

$$S(a,v,F)=\left(v-a\frac{\partial}{\partial a}-v\frac{\partial}{\partial v}+1\right)Z(a,v,F)\,,\qquad Z(v)=\sqrt{v}e^{\gamma v}\Gamma(v)\,. \tag{3.30}$$

Using this equation we determine the normalisation factor K as in [7]. We can expect that for $u\to 0$ this action should reduce to the DBI action with the gauge field F and with the tachyon potential

$$S(T,F)=T_{2p-1}\int d^{2p}xe^{-T}\sqrt{\det(\delta_{ab}+2\pi\alpha' F_{ab})}+O(\partial T)\,,\qquad T_{2p-1}=\frac{2\pi}{(4\pi^2\alpha')^p}\,. \tag{3.31}$$

In order to determine the factor K we evaluate this action on the tachyon profile $T=a+u(x^1)^2$ and compare this result with the partition function (3.29) in the limit $u\to 0$. The previous expression gives

$$\begin{aligned}S(T,F)&=T_{2p-1}\int d^{2p-1}x\sqrt{\det(\delta_{ab}+2\pi\alpha' F_{ab})}e^{-a}\int e^{-u(x^1)^2}dx^1\\&=T_{2p-1}\int d^{2p-1}xe^{-a}\sqrt{\det(\delta_{ab}+2\pi\alpha' F_{ab})}\sqrt{\frac{\pi}{u}}\,,\end{aligned} \tag{3.32}$$

and when we compare this result with (3.29) evaluated for small u we obtain

$$\begin{aligned}Z(a,v,F)&=K\int d^{2p-1}xe^{-a}\sqrt{\det(\delta_{ij}+2\pi\alpha' F_{ij})}\frac{1}{\sqrt{v}}\\&=K\int d^{2p-1}xe^{-a}\sqrt{\det(\delta_{ij}+2\pi\alpha' F_{ij})}\sqrt{1+(2\pi\alpha'\theta)^2}\frac{1}{\sqrt{2\alpha' u}}\\&\Rightarrow K=T_{2p-1}\sqrt{2\pi\alpha'}\,.\end{aligned} \tag{3.33}$$

Now we are ready to calculate the tension of the resulting D$(2p-2)$-brane, following [7]. Variation of this action with respect to a gives

$$\frac{\delta S}{\delta a}=\left[v+a-v\frac{\partial}{\partial v}\right]Z(v)=0\Rightarrow a^\star=-v+v\frac{d\ln Z(v)}{dv}\,, \tag{3.34}$$

and the action evaluated with this value of a^* is

$$\begin{aligned}S(a^\star,v,F)&=\exp(\Theta(v))K\int d^{2p-1}x\sqrt{\det(\delta_{ij}+2\pi\alpha' F_{ij})}\,,\\\Theta(x)&=x-x\frac{d\ln Z(x)}{dx}+\ln Z(x)\,.\end{aligned} \tag{3.35}$$

As was shown in [2, 7] the minimum of $\Theta(x)$ corresponds to x going to infinity. In this limit we have [7]

$$\Theta(x) = \log\sqrt{2\pi} + \frac{1}{6x} + O\left(\frac{1}{x^2}\right), x \to \infty\,, \tag{3.36}$$

which gives the value of the action

$$S = T_{2p-1}\sqrt{4\pi^2\alpha'}\int d^{2p-1}x\sqrt{\det(\delta_{ij}+2\pi\alpha' F_{ij})}\,. \tag{3.37}$$

With using $T_{2p-1}\sqrt{4\pi^2\alpha'} = T_{2p-2}$ we see that the previous expression is an exact value of the action for D$(2p-2)$-brane with the background gauge field F_{ij}. Note that this action does not depend on the gauge field F_{xy}.

We have seen in this section that from the configurations of N D-instantons in the limit $N \to \infty$ we obtain all D-branes in the bosonic string theory. We have analysed this system in the framework of BSFT which again leads to the correct result as in [7]. In the next section we extend this analysis to the case of non-BPS and BPS D-branes in type-IIA theory. Extension to the type-IIB theory is straightforward, we simply start with the configuration of unstable D0-branes.

4. D-instantons in the superstring boundary string field theory

In this section we would like to analyse the same problem in the context of the superstring boundary string field theory. In this case the calculation is simpler since as was conjectured in [11][4] the space-time action S is equal to the disk partition sum

$$S(\lambda_I) = Z(\lambda_I)\,, Z(\lambda_I) = \langle e^{-S_{\mathrm{bound}}(\lambda_I)}\rangle\,. \tag{4.1}$$

The bulk action is

$$S_{\mathrm{Bulk}} = \frac{1}{4\pi}\int d^2z\left(\frac{2}{\alpha'}\partial X^{\mu}\partial X_{\mu} + \psi^{\mu}\overline{\partial}\psi_{\mu} + \tilde{\psi}^{\mu}\partial\tilde{\psi}_{\mu}\right)\,. \tag{4.2}$$

The mode expansion at the boundary of the disk is [18] (in NS sector)

$$\psi^{\mu}(\tau) = \sum_{r=1/2}^{\infty}\left(\psi^{\mu}_{r}e^{ir\tau} + \psi^{\mu}_{-r}e^{-ir\tau}\right)\,. \tag{4.3}$$

and the bulk action

$$S_{\mathrm{Bulk}} = \frac{1}{2}\sum_{n=1}^{\infty} nX^{\mu}_{-n}X^{\nu}_{n}\delta_{\mu\nu} + i\sum_{r=1/2}^{\infty} r\psi^{\mu}_{-r}\psi^{\nu}_{r}\delta_{\mu\nu}\,, \qquad \mu,\nu = 1,\ldots,10\,. \tag{4.4}$$

[4] Similar approach to this problem has also been discussed in [21] where many references on the relevant papers can be found.

The next step is to introduce the boundary interactions which should be invariant under supersymmetry transformations. This can be done by working in boundary superspace [11, 18, 19, 20, 21] with coordinates $\hat{\tau} = (\tau, \theta)$ and

$$\mathbf{X}^\mu = X^\mu + \sqrt{\alpha'}\theta\psi^\mu, \ D = \partial_\theta + \theta\partial_\tau \,. \tag{4.5}$$

The simplest example of supersymmetric boundary action corresponds to gauge field. For abelian gauge field the supersymmetric boundary action is

$$\begin{aligned} -i\int d\tau d\theta A_\mu(\mathbf{X})D\mathbf{X}^\mu &= -i\int d\tau d\theta \left(A_\mu(X) + \partial_\nu A_\mu(X)\sqrt{\alpha'}\theta\psi^\nu\right)\left(\sqrt{\alpha'}\psi^\mu + \theta\partial_\tau X^\mu\right) \\ &= -i\int d\tau \left(A_\mu(X(\tau))\partial_\tau X^\mu + \frac{1}{2}\alpha' F_{\mu\nu}\psi^\mu\psi^\nu\right), \end{aligned} \tag{4.6}$$

where we have used

$$\theta^2 = 0\,, \qquad \int d\theta = 0\,, \qquad \int d\theta\theta = 1\,. \tag{4.7}$$

In case of Dirichlet boundary conditions for x^a, $a, b = p+1, \dots, 10$ and Neumann boundary conditions for x^μ, $\mu, \nu = 1, \dots, p$ we will write the boundary action as

$$\begin{aligned} -i\int d\tau d\theta &\left(A_\mu(\mathbf{X}^\mu)D\mathbf{X}^\mu + \Phi_a(\mathbf{X}^\mu)\tilde{D}\mathbf{X}^a\right) = \\ &= -i\int d\tau \left(A_\mu(X^\mu(\tau))\partial_\tau X^\mu(\tau) + \frac{1}{2}\alpha' F_{\mu\nu}\psi^\mu(\tau)\psi^\nu(\tau)\right) - \\ &\quad -i\int d\tau\left(\Phi_a(X^\mu(\tau))\partial_n X^a(\tau) + \alpha'\partial_\mu\Phi_a(X^\mu(\tau))\psi^\mu(\tau)\psi^a(\tau)\right), \end{aligned} \tag{4.8}$$

where we have used the fact that the background fields are functions of the string fields $X^a(\tau)$ obeying Neumann boundary conditions and we have defined

$$\tilde{D} = \partial_\theta + \theta\partial_n \,. \tag{4.9}$$

Non-abelian extension is given (In case of all Neumann boundary conditions, generalisation to the case of lower dimensional D-branes is straightforward as we will see in case of D-instantons.)

$$e^{-S_A} = \mathrm{Tr}\,\hat{P}e^{i\int d\tau d\theta A_\mu(\mathbf{X})D\mathbf{X}^\mu}\,, \tag{4.10}$$

where the symbol $\hat{P}$ is defined as

$$\hat{P}e^{\int d\hat{\tau}M(\hat{\tau})} = \sum_{N=0}^{\infty}\int d\hat{\tau}_1\cdots d\hat{\tau}_N\Theta(\hat{\tau}_{12})\Theta(\hat{\tau}_{23})\cdots\Theta(\hat{\tau}_{N-1,N})M(\hat{\tau}_1)M(\hat{\tau}_2)\cdots M(\hat{\tau}_N)\,, \tag{4.11}$$

where $d\hat{\tau} = d\tau d\theta$, $\hat{\tau}_{12} = \tau_1 - \tau_2 - \theta_1\theta_2$ and Θ is a step function whose expansion is equal to $\Theta(\hat{\tau}_1 - \hat{\tau}_2) = \theta(\tau_1 - \tau_2) - \delta(\tau_1 - \tau_2)\theta_1\theta_2$. As was shown in [20], these contact terms are essential for world-sheet supersymmetry and they are also crucial for gauge invariance as they contribute with the $[A_\mu, A_\nu]$ in the field strength. More precisely, the integration over θ gives the result

$$e^{-S_A} = \mathrm{Tr}\, P e^{i\int d\tau [A_\mu(X)\partial_\tau A^\mu + \frac{\alpha'}{2} F_{\mu\nu}\psi^\mu\psi^\nu]}\,, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]\,. \tag{4.12}$$

It is easy to generalise this boundary interaction to the case of lower dimensional Dp-branes. We will mainly consider D-instantons and for them the boundary interaction is

$$\begin{aligned} e^{-S_A} &= \mathrm{Tr}\, \hat{P} \exp\left(i\int d\hat{\tau}\Phi_I \tilde{D}\mathbf{X}^I\right) \\ &= \mathrm{Tr}\, P \exp\left(i\int d\tau \left[\Phi_I \partial_n X^I(\tau) - i\frac{\alpha'}{2}[\Phi_I, \Phi_J]\psi^I(\tau)\psi^J(\tau)\right]\right), \end{aligned} \tag{4.13}$$

with $I, J = 1, \ldots, 10$. As in bosonic case, the description of even dimensional D-branes is very straightforward and corresponds to the natural emergence of these D-branes in the matrix models [47, 48, 50, 52, 53]. Let us consider the background configuration in the form

$$[\Phi_a, \Phi_b] = i\theta_{ab}\,, \quad a = 1, \ldots, 2p\,, \qquad \Phi_\alpha = 0\,, \quad \alpha = 2p+1, \ldots, 10\,. \tag{4.14}$$

Then the fermionic term in (4.13) is equal to

$$-i\frac{\alpha'}{2}[\Phi_a, \Phi_b]\psi^a(\tau)\psi^b(\tau) = \frac{\alpha'}{2}\theta_{ab}\psi^a(\tau)\psi^b(\tau)1_{N\times N}\,. \tag{4.15}$$

We see that this term is proportional to the unit matrix so that the following expression can be taken out the trace

$$\exp\left(i\int d\tau \frac{\alpha'}{2}\theta_{ab}\psi^a(\tau)\psi^b(\tau)\right)\,. \tag{4.16}$$

Now it is easy to see, following the calculations presented in section (3), that this D-instantons configurations describe D$(2p-1)$-brane with the background gauge field $F_{ab} = \theta_{ab}$ with the boundary action

$$\begin{aligned} e^{-S_A} &= \exp\left(i\int d\tau \left[A_a(X)\partial_\tau X^a(\tau) + \frac{\alpha'}{2}F_{ab}\psi^a(\tau)\psi^b(\tau)\right]\right), \\ F_{ab} &= \partial_a A_b - \partial_b A_a = \theta_{ab}\,. \end{aligned} \tag{4.17}$$

Consequently the action evaluated on this particular D-instanton background is equal to

$$S(F) = Z(F) = \langle e^{-S_A}\rangle = \sqrt{2}T_{2p-1}\int d^{2p}x\sqrt{\det(\delta_{ab} + 2\pi\alpha' F_{ab})}\,. \tag{4.18}$$

In order to describe odd dimensional D-brane we should include the tachyon in the boundary action and calculate its condensation. The inclusion of tachyon into the boundary interaction in supersymmetric case has been done in [11, 18, 21]. We introduce superfield $\Gamma = \xi + \theta F$ living on the boundary of the disk. Now the tachyon boundary action is (For all $\mathbf{X}(\tau)$ obeying Neumann boundary conditions)

$$e^{-S_{\mathrm{bound}}} = \mathrm{Tr}\,\hat{P}\exp\left(\int \frac{d\hat{\tau}}{2\pi}(\Gamma D\Gamma + T(\mathbf{X}))\Gamma\right). \tag{4.19}$$

In abelian case we can easily perform the integration over θ. When we also integrate out the auxiliary field Γ we obtain [11]

$$e^{-S_{\mathrm{bound}}} = \exp\left(-\frac{1}{4}\int\frac{d\tau}{2\pi}\left[T(X)^2 + (\psi^\mu\partial_\mu T)\frac{1}{\partial_\tau}(\psi^\nu\partial_\nu T)\right]\right). \tag{4.20}$$

The generalisation of this action to the case of N D-branes with Neumann boundary conditions in x^a, $a = 1,\ldots,k$ and Dirichlet boundary conditions in x^i, $i = k+1,\ldots,10$ is

$$\begin{aligned} S(\Phi,T) &= Z(\Phi,T) = \langle e^{-S_A - S_{\mathrm{bound}}}\rangle \\ &= \left\langle \mathrm{Tr}\,\hat{P}\exp\left(\int d\hat{\tau}\left[\frac{1}{2\pi}\Gamma D\Gamma + \frac{1}{2\pi}T(\mathbf{X}^a)\Gamma + \right.\right.\right. \\ &\qquad\qquad \left.\left.\left. + iA_a(\mathbf{X}^a)D\mathbf{X}^a + i\Phi_i(\mathbf{X}^a)\tilde{D}X^i\right]\right)\right\rangle. \end{aligned} \tag{4.21}$$

In the following we will consider N D(-1)-branes with the boundary interaction

$$e^{-S_{\mathrm{bound}}-S_A} = \mathrm{Tr}\,\hat{P}\exp\left(\int d\hat{\tau}\left[\frac{1}{2\pi}\Gamma D\Gamma + \frac{1}{2\pi}T\Gamma + i\Phi_I\tilde{D}\mathbf{X}^I\right]\right), \qquad I = 1,\ldots,10\,. \tag{4.22}$$

The basic idea is the same as in previous section (3). We start with performing the integration over θ. In order do that, we express the previous expression using the elegant formalism [21] in which the boundary action has a form

$$e^{-S_A - S_{\mathrm{bound}}} = \int[d\hat{\eta}][d\hat{\overline{\eta}}]\exp\left(\int d\hat{\tau}\left[\hat{\overline{\eta}}_b D\hat{\eta}^b + \frac{1}{2\pi}\Gamma D\Gamma + \hat{\overline{\eta}}_a\left[\frac{1}{2\pi}T^a_b\Gamma - i\Phi^a_{bI}\tilde{D}\mathbf{X}^I\right]\hat{\eta}^b\right]\right), \tag{4.23}$$

where the fields $\hat{\eta}^a = \eta^a + \theta\chi^a$, $\hat{\overline{\eta}}_a = \overline{\eta}_a + \theta\chi_a$ transform in antifundamental and fundamental representation of U(N). The integration over θ gives

$$\begin{aligned} S_{\mathrm{bound}} + S_A = I = \int d\tau\Big(&\overline{\eta}_a\dot{\eta}^a - \overline{\chi}_a\chi^a + \frac{1}{2\pi}\xi\dot{\xi} - \frac{1}{2\pi}F^2 + \\ &+ \frac{1}{2\pi}\left(-\overline{\eta}_a T^a_b\xi\chi^b + \overline{\eta}_a T^a_b F\eta^b - \overline{\chi}_a T^a_b\xi\eta^b\right) + \\ &+ i\overline{\eta}_a\Phi^a_{bI}\sqrt{\alpha'}\psi^I\chi^b - i\overline{\eta}_a\Phi^a_{bI}\partial_n X^I\eta^b + i\overline{\chi}_a\Phi^a_{bI}\sqrt{\alpha'}\psi^I\eta^b\Big), \end{aligned} \tag{4.24}$$

In the previous expression $a, b = 1, \ldots, N$ are matrix indices, $I = 1, \ldots, 10$ are space-time indices. As a next thing we integrate out the auxiliary field $\chi^a, \overline{\chi}_a$ which gives

$$
\begin{aligned}
I = \int d\tau \Big(& \overline{\eta}_a \dot{\eta}^a + \frac{1}{2\pi}\xi\dot{\xi} - \frac{1}{2\pi}F^2 + \\
& + \overline{\eta}_a \left[\frac{i\sqrt{\alpha'}}{2\pi}\xi\psi^I[\Phi_I, T]^a_b + \frac{1}{2\pi}T^a_b F - i\Phi^a_{bI}\partial_n X^I - \frac{\alpha'}{2}[\Phi_I, \Phi_J]^a_b \psi^I\psi^J \right]\eta^b \Big) .
\end{aligned}
\tag{4.25}
$$

We rewrite the path integral over $\overline{\eta}, \eta$ in (4.25) as a path-ordered trace so that we obtain

$$
\begin{aligned}
\langle e^{-S_{\rm bound} - S_A} \rangle = \Big\langle \mathrm{Tr}\, P \exp \Big(\int d\tau \Big[& -\frac{1}{2\pi}\xi\dot{\xi} + \frac{1}{2\pi}F^2 + \frac{i\sqrt{\alpha'}}{2\pi}\xi\psi^I[\Phi_I, T] - \\
& - \frac{1}{2\pi}TF + i\Phi_I\partial_n X^I + \frac{\alpha'}{2}[\Phi_I, \Phi_J]\psi^I\psi^J \Big] \Big) \Big\rangle .
\end{aligned}
\tag{4.26}
$$

Let us consider the ansatz

$$
T = u\Phi_2 \, , \qquad [\Phi_x, \Phi_y] = i\epsilon_{xy}\theta \, , \quad x, y = 1, 2 \, , \qquad [\Phi_i, \Phi_j] = i\theta_{ij}, \quad i, j = 3, \ldots, 2p \, , \tag{4.27}
$$

so that θ in $[\Phi_a, \Phi_b] = i\theta_{ab}$ has a form

$$
\theta = \begin{pmatrix} 0 & \theta & 0 \\ -\theta & 0 & 0 \\ 0 & 0 & \theta_{ij} \end{pmatrix} , \qquad i, j = 3, \ldots, 2p \, . \tag{4.28}
$$

Then the previous expression is equal to

$$
\begin{aligned}
\langle e^{-S_{\rm bound} - S_A} \rangle = \Big\langle \int [d\phi_a] \exp \Big(\int d\tau \Big[& \frac{1}{2} i\phi_a\theta^{ab}\dot{\phi}_b - \frac{1}{2\pi}\xi\dot{\xi} + \frac{1}{2\pi}F^2 + \frac{\sqrt{\alpha'}}{2\pi}\xi\psi^1 u\theta - \\
& - \frac{1}{2\pi}u\phi_2 F + i\phi_a\partial_n X^a + \frac{i\alpha'}{2}\theta_{ab}\psi^a\psi^b \Big] \Big) \Big\rangle .
\end{aligned}
\tag{4.29}
$$

The integration over auxiliary field F gives

$$
F(\tau) = \frac{1}{2}u\phi_2(\tau) \tag{4.30}
$$

and we have

$$
\begin{aligned}
\langle e^{-S_{\rm bound} - S_A} \rangle = \Big\langle \int [d\phi_a] \exp \Big(\int d\tau \Big[& i\frac{1}{2}\phi_a\theta^{ab}\dot{\phi}_b - \frac{1}{2\pi}\xi\dot{\xi} + \frac{\sqrt{\alpha'}}{2\pi}\xi\psi^1 u\theta - \frac{1}{8\pi}u^2(\phi_2)^2 + \\
& + i\phi_a\partial_n X^a + \frac{i\alpha'}{2}\theta_{ab}\psi^a\psi^b \Big] \Big) \Big\rangle .
\end{aligned}
\tag{4.31}
$$

We can take out the following expression from the path integral over ϕ

$$\exp\left(\int d\tau\left[-\frac{1}{2\pi}\xi\dot{\xi}+\frac{\sqrt{\alpha'}}{2\pi}\xi\psi^1 u\theta+\frac{i\alpha'}{2}\theta_{ab}\psi^a\psi^b\right]\right). \tag{4.32}$$

Next calculation is the same as in previous section (3). The $i,j=3,\dots,2p$ components give the same result as in the case of even dimensional D-brane without exciting tachyon and the integration over $x,y=1,2$ gives

$$\begin{aligned}&\exp\left(-\frac{1}{2}\int d\tau d\tau'\partial_n X^x(\tau)\Delta(\tau-\tau')^{-1}_{xy}\partial_n X^y(\tau')\right),\\ &\Delta(\tau-\tau')^{-1}_{xy}=\sum_n (E_n)^{-1}_{xy}e^{in(\tau-\tau')}, \qquad (E_n^{-1})_{xy}=\frac{\theta^2}{n^2}\begin{pmatrix}-\frac{1}{4\pi}u^2 & -\frac{n}{\theta}\\ \frac{n}{\theta} & 0\end{pmatrix},\end{aligned} \tag{4.33}$$

which together with the F_{ij} and F_{xy} terms gives the result

$$\begin{aligned}Z=\Big\langle\exp\Big(\int d\tau\Big[&-\frac{1}{2\pi}\xi\dot{\xi}+\frac{\sqrt{\alpha'}}{2\pi}\xi\psi^1 u-\frac{1}{8\pi}u^2(X^1)^2+\\ &+iA_a(X^a)\partial_\tau X^a+\frac{i\alpha'}{2}\theta_{ab}\psi^a\psi^b\Big]\Big)\Big\rangle,\end{aligned} \tag{4.34}$$

where we have made replacement $u\theta\to u$. We can also integrate out the auxiliary field ξ which leads to

$$\dot{\xi}(\tau)=\frac{u\sqrt{\alpha'}}{2}\psi^1(\tau)\Rightarrow\xi(\tau)=\frac{u\sqrt{\alpha'}}{2}\frac{1}{\partial_\tau}\psi^1(\tau) \tag{4.35}$$

and we obtain the same expression as in [11, 12]

$$\begin{aligned}\langle e^{-S_A-S_{\mathrm{bound}}}\rangle=\Big\langle\exp\Big(\int d\tau\Big[&-\frac{1}{8\pi}(u^2(X^1)^2+\alpha' u^2\psi^1\frac{1}{\partial_\tau}\psi^1)+\\ &+iA_a(X^a)\partial_\tau X^a+\frac{i\alpha'}{2}F_{ab}\psi^a\psi^b\Big]\Big)\Big\rangle, \qquad F_{ab}=\theta_{ab}.\end{aligned} \tag{4.36}$$

Then it is easy to see that the tachyon condensation really leads to the emergence of odd dimensional D-brane exactly in the same way as in [11]. The partition function is equal to [11, 12, 15]

$$Z=K\int d^{2p-1}xZ(a,v)_{\mathrm{Fermi}}\sqrt{\det(\delta_{ij}+2\pi\alpha' F_{ij})}, \qquad v=\frac{\alpha' u^2/2}{1+(2\pi\alpha'\theta)^2}. \tag{4.37}$$

with [11]

$$Z(a,v)_{\mathrm{Fermi}}=4^v\frac{Z_1(v)^2}{Z_1(2v)}, \qquad Z_1(v)=\sqrt{v}e^{\gamma v}\Gamma(v), \tag{4.38}$$

and where K is a numerical factor that will be determined as in section (3). When we calculate the partition sum for constant tachyon $T=a1_{N\times N}$ we obtain the exact tachyon potential $e^{-a^2/4}$ multiplied with the DBI term $\sqrt{\det(\delta_{ab} + 2\pi\alpha' F_{ab})}$ arising from the partition sum calculated in the pure gauge field background. Then we can expect that for slowly varying tachyon field the action corresponds to the non-BPS D$(2p-1)$-brane action

$$S = \sqrt{2}T_{2p-1} \int d^{2p}x e^{-T^2/4} \sqrt{(1 + (2\pi\alpha'\theta)^2)\det(\delta_{ij} + 2\pi\alpha' F_{ij})} + O(\partial T)\,, \tag{4.39}$$

This action evaluated on the tachyon profile $T = ux^1$ should be equal to the partition sum in the limit $u \to 0$. From this requirement we can determine the overall normalisation constant in the partition sum as well as the normalisation of the tachyon kinetic term [11]. However for our purposes we can only consider the normalisation constant K. Then the action (4.39) evaluated on the tachyon profile $T(x) = ux^1$ is equal to

$$\begin{aligned} &\sqrt{2}T_{2p-1} \int d^{2p-1}x \sqrt{(1 + (2\pi\alpha'\theta)^2)\det(\delta_{ij} + 2\pi\alpha' F_{ij})} \int dx^1 e^{-u^2(x^1)^2/4} = \\ &\qquad = 2\frac{\sqrt{\pi}}{u}\sqrt{2}T_{2p-1} \int d^{2p-1}x \sqrt{(1 + (2\pi\alpha'\theta)^2)\det(\delta_{ij} + 2\pi\alpha' F_{ij})}\,. \end{aligned} \tag{4.40}$$

On the other hand, in the limit $u \to 0$ we have

$$Z(v)_{\mathrm{Fermi}} = \sqrt{\frac{2}{v}} + O(v)\,, \qquad v \sim 0\,, \tag{4.41}$$

so that the partition function is equal to

$$Z = K\frac{\sqrt{2}\sqrt{2}}{\sqrt{\alpha' u^2}} \int d^{2p-1}x \sqrt{(1 + (2\pi\alpha'\theta)^2)\det(\delta_{ij} + 2\pi\alpha' F_{ij})} \tag{4.42}$$

and consequently

$$K = \sqrt{2}T_{2p-1}\sqrt{\pi\alpha'}\,. \tag{4.43}$$

Using this result it is easy to see that the action arising from the tachyon condensation is equal to (In this case the tachyon condensation corresponds to $u \to \infty$ [11])

$$\begin{aligned} S &= Z(\infty) = \sqrt{2\pi\alpha'}T_{2p-1}\sqrt{2\pi} \int d^{2p-1}x \sqrt{\det(\delta_{ij} + 2\pi\alpha' F_{ij})} = \\ &= T_{2p-2} \int d^{2p-1}x \sqrt{\det(\delta_{ij} + 2\pi\alpha' F_{ij})}\,, \end{aligned} \tag{4.44}$$

where we have used

$$Z(v) \sim \sqrt{2\pi} + O(v^{-1})\,, \qquad u \to \infty\,. \tag{4.45}$$

The result (4.44) is a correct value of the action for D$(2p-2)$-brane with the background gauge field strength F_{ij}.

In this section we have studied the emergence of all D-branes in type-IIA theory from the configurations of infinite many non-BPS D(-1)-instantons. Exactly in the same way we could proceed with the infinite many non-BPS D0-branes in type-IIB theory.

5. Conclusion

In this paper we have tried to show that all D-branes in bosonic, type-IIA and type-IIB theory can emerge from configurations of infinite many D-instantons, in case of type-IIB theory from infinite many D0-branes. We have studied this system from the point of view of BSFT and we have shown that we can very easily obtain correct values of the tensions of all D-branes.

However, many open questions remain. Firstly, it would be nice to obtain non-abelian action for D-instantons, or more generally for all non-abelian D-branes from the BSFT theory. It seems that this can be easily done in the case of supersymmetric string theory at least in some special cases [51, 59]. We hope to return to these questions in the future. It would be also interesting to study the D-brane anti-D-brane system in the framework suggested in this paper, following the general construction [18]. We hope to return to this problem in the forthcoming publication.

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